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Conformal couplings and "azimuthal matching" of QCD pomerons

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Abstract. Using the asymptotic conformal invariance of perturbative QCD we derive the expression of the coupling of external states to all conformal spin p components of the forward elastic amplitude. Using the wave function formalism for structure functions at small x, we derive the perturbative coupling of the virtual photon for $p = 1$, which is maximal for linear transverse polarization. The non-perturbative coupling to the proton is discussed in terms of "azimuthal matching" between the proton color dipoles and the $q\bar{q}$ configurations of the photon. As an application, the recent conjecture of a second QCD pomeron related to the conformal spin-1 component is shown to rely upon a strong azimuthal matching of the $p = 1$ component in γ^*–proton scattering.

1 Conformal invariance of the BFKL equation

As is well known, the equation written by Balitskii, Fadin, Kuraev and Lipatov (BFKL) [1] expresses the elastic amplitude of two off-shell gluons in the high energy limit corresponding to the perturbative QCD resummation of the leading logarithms. In terms of transverse coordinates (Fourier transforms of the four external gluon transverse momenta), the equation can be schematically written $\partial f/\partial Y(k, k', q; Y) = K \otimes f$, where Y (in the case of structure functions $Y = \log 1/x_{b_j}$) is the whole rapidity range, k, k' the two-dimensional initial gluon momenta and q the 2-momentum transfer. The BFKL integro-differential kernel K obtained at leading log order is known to possess a global conformal $SL(2, \mathbb{C})$ invariance [2]. The BFKL derivation is made in the framework of the leading log approximation but it is interesting to investigate the more general consequences of the asymptotic conformal invariance. Deviations from asymptotic conformal invariance could also be studied by comparison with the results obtained with this assumption[1].

The solution of the BFKL equation is for the 4-point gluon amplitude. For practical application to the proton structure functions, say, the conformal couplings of the BFKL solution with the $q\bar{q}$ states of the virtual photon and with the proton have to be made explicit. This is the main purpose of our paper: to formulate the most general coupling of the external states and discuss the constraints imposed by the conformal symmetry of the BFKL equation.

The conformal symmetry of the BFKL equation [2] is a powerful tool. Knowing that the kernel K is invariant under the $SL(2, \mathbb{C})$ transformations, it is possible [2,4] to exactly solve the BFKL equation by expanding over the $SL(2, \mathbb{C})$ unitary irreducible representations, which are labelled by two quantum numbers, namely the "conformal dimension" $\gamma = (1/2) + i\nu$ and the "conformal spin"[2] $p \in \mathbb{N}$. In the appropriate eigenbasis K is diagonal with eigenvalues

$$\epsilon(p, \gamma) = \bar{\alpha}\chi_p(\gamma), \tag{1}$$

where $\bar{\alpha} = \alpha N_c/\pi$ and

$$\begin{aligned}\chi_p(\gamma) &= 2\Psi(1) - \Psi(p + 1 - \gamma) - \Psi(p + \gamma) \qquad (2)\\ &= \sum_{m=0}^{\infty} \left\{ \frac{1}{m + \gamma + p} + \frac{1}{p + 1 - \gamma + m} - \frac{2}{m + 1} \right\}.\end{aligned}$$

Using the expansion over the whole conformal basis leads to an expression for the structure function:

$$\begin{aligned}F_2(Y, Q^2) &= \sum_p F_p(Y, Q^2) \qquad (3)\\ &\equiv \sum_p \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} d\gamma \left(\frac{Q}{Q_0}\right)^{2\gamma} e^{\overline{\alpha}\chi_p(\gamma)Y} f_p(\gamma),\end{aligned}$$

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[1] Throughout the present paper, we will stick to the conformal properties of the BFKL equation. However conformal invariance could be partly preserved at next-leading order, or it could be realized as an approximate conformal invariance [3]. The tools developed in our paper could then be extended to this case.

[2] The conformal spin ($p = n/2, n \in \mathbb{Z}$, in the notation of [2]) can be half-integer, but only integer values contribute to the structure functions [5]. It takes also negative values, but the decomposition over positive eigenvalues is complete and thus sufficient to describe the conformal expansion.

where $f_p(\gamma)$ is obtained from the couplings of the different conformal spin components to the external sources. The aim of our paper is to discuss these functions $f_p(\gamma)$ taking into account the constraints due to conformal invariance.

In the expression (3), one usually sticks to the component $p = 0$ which gives rise to the “hard” QCD pomeron in the leading order BFKL formalism. In phenomenological applications, the perturbative coupling of the conformal component $p = 0$ to the virtual photon has been known since a long time [6–9] and some models of the non-perturbative coupling to the proton have been discussed [9].

However, little has been done on higher conformal spins. They have been considered in two-jet production with a large rapidity interval in hadron–hadron collisions [10] and in the forward jet production in deep inelastic scattering [11], which correspond to two “hard” vertices with similar characteristic scales. We shall come back to the corresponding perturbative QCD calculations later on in the discussion. More recently, the general conformal coupling has been formally derived in the eikonal approximation [12], leading to interesting selection rules. But higher spin components were expected to have no practical applications at a high energy (rapidity interval) since they are at first sight power suppressed in energy. This is indeed the case for the processes considered in [10,11].

However, recently it has been noticed [13] that the spin component $p = 1$ may have a non-negligible impact for processes corresponding to vertices with different characteristic scales and in particular for proton structure functions at moderate and large Q^2. This is due to a “sliding” mechanism which shifts its effective intercept up and thus drastically changes the energy dependence. The $p = 1$ spin component may even be interpreted as the remnant of the well-known “soft” pomeron in the high Q^2 region. This result is to be put in perspective with the two-pomeron conjecture of [14], where the “soft” pomeron is considered to be of the higher-twist type, while the “hard” pomeron would represent some kind of leading twist[3]. Hence, it is worth studying in detail the constraints and properties of conformal couplings to QCD pomerons, both from a perturbative (for the virtual photon) and non-perturbative (for the proton) points of view.

Section 2 is devoted to the general formalism for the coupling to a generic conformal spin component of the BFKL solution. In Sect. 3, we derive the perturbative coupling to the virtual photon wave function in terms of its $q\bar{q}$ configurations and introduce a class of models for the non-perturbative couplings to the proton satisfying appropriate constraints. Then, in Sect. 4, we make a phenomenological application to the two-pomeron conjecture based on conformal spin components of the proton structure functions, which leads to the necessity of a strong azimuthal “matching” condition which is discussed in detail. A summary and our conclusions are presented in Sect. 5.

[3] Note however that [14] is written in the conventional Regge formalism while the study of [13] is made in the framework of the BFKL equation and its conformal invariant setting.

2 Conformal impact factors

Following [2] the virtual photon–proton elastic BFKL scattering amplitude reads

$$A(s, -q^2) = \mathrm{i}s \int \frac{\mathrm{d}\omega}{2\mathrm{i}\pi} \left(\frac{s}{Q^2}\right)^{\omega} f_\omega(q^2), \tag{4}$$

where $s/Q^2 \approx 1/x$, q^2 is the four-momentum transfer squared and

$$f_\omega(q^2) = \int \mathrm{d}^2k\mathrm{d}^2k' \mathcal{V}^{(1)}(k,q)\overline{\mathcal{V}}^{(2)}(k',q) f_\omega(k,k',q). \tag{5}$$

$f_\omega(k,k',q)$ is nothing else than the $Y \to \omega$ Mellin transformed of the two-gluon elastic amplitude verifying the BFKL evolution equation (see Sect. 1). $\mathcal{V}^{(1)}(k,q)$ and $\mathcal{V}^{(2)}(k',q)$ are the so-called *impact factors* describing the coupling of the initial states to the gluons.

After straightforward calculations using the conformal basis of eigenvectors [2,4], one may write

$$f_\omega(q^2) = \sum_p \int \frac{\mathrm{d}\gamma}{2\mathrm{i}\pi} \frac{c(p,\gamma)}{\omega - \epsilon(p,\gamma)} V_1^{p,\gamma}(q) \overline{V}_2^{p,\gamma}(q), \tag{6}$$

with

$$V_{1,2}^{p,\gamma}(q) = \frac{1}{(2\pi)^3} \times \int \mathrm{d}^2\rho\mathrm{d}^2\rho'\mathrm{d}^2k \mathcal{V}^{(1,2)}(k,q)\, \mathrm{e}^{\mathrm{i}k\rho + \mathrm{i}(q-k)\rho'} E^{p,\gamma}(\rho,\rho'), \tag{7}$$

where $\epsilon(p,\gamma)$ is given in (1) and $E^{p,\gamma}(\rho,\rho')$ are the $SL(2,\mathbb{C})$ eigenfunctions

$$E^{p,\gamma}(\rho,\rho') = \left(\frac{\rho-\rho'}{\rho\rho'}\right)^{\gamma-p} \left(\frac{\bar\rho-\bar\rho'}{\bar\rho\bar\rho'}\right)^{\gamma+p}, \tag{8}$$

and

$$c(p,\gamma) = \frac{\nu^2 + p^2}{\left(\nu^2 + \left(p-\frac{1}{2}\right)^2\right)\left(\nu^2 + \left(p+\frac{1}{2}\right)^2\right)}, \tag{9}$$

where $\gamma \equiv (1/2) + \mathrm{i}\nu$. In the forward direction ($q = 0$) the formula (8) simplifies. After changing variables to $\rho + \rho' = 2b$ and $\rho - \rho' = r$ and combining the relations (see [2])

$$\int \mathrm{d}^2b E^{p,\gamma}\left(b + \frac{r}{2}, b - \frac{r}{2}\right) = \frac{b_{p,\gamma}}{(2\pi)^2} r^{\gamma-p}\bar{r}^{\gamma+p}, \tag{10}$$

and (see [15])

$$\int \mathrm{d}^2u u^{\gamma-p}\bar{u}^{\gamma+p}\mathrm{e}^{(\mathrm{i}/2)(u+\bar u)} = 2\pi \int \mathrm{d}|u||u|^{1+2\gamma} J_{2p}(|u|) = 4^{1+\gamma}\pi \frac{\Gamma(\gamma+p+1)}{\Gamma(p-\gamma)}, \tag{11}$$

one gets

$$V_{1,2}^{p,\gamma}(q=0) = 2^{1+2\gamma}\frac{\Gamma(\gamma+p+1)}{\Gamma(p-\gamma)}\, b_{p,\gamma} \times \int \mathrm{d}^2k \mathcal{V}^{(1,2)}(k) k^{-(\gamma+p+1)}\bar{k}^{-(\gamma-p+1)}, \tag{12}$$

where $b_{p,\gamma}$ is a $SL(2,\mathbb{C})$ constant given in [2] and verifying $|b_{p,\gamma}|^2 = \pi^6/(p^2+\nu^2)$.

Using the relation $\Im m A(q^2=0) \equiv s\sigma_{\rm tot} = s/(4\pi Q^2) \times F_2(Y,Q^2)$, one finally obtains

$$
\begin{aligned}
F_2(x,Q^2) &\sim \sum_p \int \mathrm{d}\gamma x^{-\epsilon(p,\gamma)} \left(\frac{Q}{Q_0}\right)^{2\gamma} V_1 \overline{V}_2 \\
&= \sum_p \int \mathrm{d}\gamma \left|\frac{\Gamma(p+\gamma)}{\Gamma(p-\gamma+1)}\right|^2 x^{-\epsilon(p,\gamma)} \left(\frac{Q}{Q_0}\right)^{2\gamma} \\
&\times \int \mathrm{d}^2\kappa \mathcal{V}_1(\kappa) \kappa^{-(\gamma+p+1)} \overline{\kappa}^{-(\gamma-p+1)} \\
&\times \int \mathrm{d}^2\kappa_0 \overline{\mathcal{V}_2}(\kappa_0) \kappa_0^{-2+\gamma+p} \overline{\kappa_0}^{-2+\gamma-p}, \qquad (13)
\end{aligned}
$$

where one introduces the natural scaling variables $k/Q = \kappa$ for the (photon) vertex V_1 and $k_0/Q_0 = \kappa_0$ for the (proton) vertex V_2. Note that the Gamma function prefactors boil down to a factor 1 on the integration line over the imaginary axis $\gamma = (1/2) + \mathrm{i}\nu$.

Let us consider for instance the first components ($p = 0, 1$). By separation of modulus and azimuthal integration over κ, they correspond to the two first coefficients of the Fourier expansion

$$
\mathcal{V}_{1,2}(\kappa) = \alpha_{1,2}(|\kappa|) + \beta_{1,2}(|\kappa|)\cos(2\varphi) + \ldots \qquad (14)
$$

In the case of proton structure functions and specializing to the two first components, one obtains

$$
\begin{aligned}
F_2(x,Q^2) &\sim \int \mathrm{d}\gamma x^{-2\bar{\alpha}(\Psi(1)-\mathrm{Re}\Psi(\gamma))} \left(\frac{Q}{Q_0}\right)^{2\gamma} f_0(\gamma) \\
&+ \int \mathrm{d}\gamma x^{-2\bar{\alpha}(\Psi(1)-\mathrm{Re}\Psi(\gamma+1))} \left(\frac{Q}{Q_0}\right)^{2\gamma} f_1(\gamma) \\
&+ \sum_{p\geq 2} \int \mathrm{d}\gamma \ldots, \qquad (15)
\end{aligned}
$$

with

$$
\begin{aligned}
f_0(\gamma) &= \int_0^\infty \mathrm{d}|\kappa||\kappa|^{-1-2\gamma} \alpha_1(|\kappa|) \\
&\times \int_0^\infty \mathrm{d}|\kappa_0||\kappa_0|^{-3+2\gamma} \alpha_2(|\kappa_0|). \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
f_1(\gamma) &= \int_0^\infty \mathrm{d}|\kappa||\kappa|^{-1-2\gamma} \beta_1(|\kappa|) \\
&\times \int_0^\infty \mathrm{d}|\kappa_0||\kappa_0|^{-3+2\gamma} \beta_2(|\kappa_0|). \qquad (17)
\end{aligned}
$$

Note a positivity constraint in the case of the eikonal coupling for which [12]

$$
\mathcal{V}(\kappa) \propto 4 \int \mathrm{d}^2 r \Phi(r) \sin^2(\kappa r/2)
$$

where $\Phi(r)$ is the probability distribution of the $q\bar{q}$ configurations in coordinate space . Hence, a positivity condition $\mathcal{V}(\kappa) > 0$ holds which leads to $|\beta| < \alpha$. However β can be negative as is indeed the case in some processes like forward jet production in DIS [11]. Note also that the positivity constraint does not hold if there are not only $q\bar{q}$ configurations in the Fock space of the target (e.g. for the proton).

3 Conformal couplings to $q\bar{q}$ configurations

Let us first derive the conformal couplings to the virtual photon. In the perturbative QCD framework and for the $p = 0$ component, it is possible to derive the couplings from first order (virtual) gluon–(virtual) photon fusion graphs, thanks to the k_T-factorization property [7]. Our aim is to start from these results and derive the corresponding coupling to higher spin components. In fact, for the reason of the spin being 1 for the virtual photon, only the conformal spin $p \leq 1$ can be obtained from the polarization components of the photon [18]. Indeed, the only way to get the $p = 1$ component is an interference term between the transversely polarized components with helicity ± 1 as we shall see now.

Interestingly, the factorization properties of QCD in the high energy regime can be put into two equivalent forms [9]. As sketched in Fig. 1, the perturbative[4] coupling of the virtual photon to a dipole can be described by two different factorized formulae. One way is to use the k_T-factorization property [7] which relates the γ^*–dipole cross-section to the product of the impact factors V by a g^*–dipole cross-section where g^* is an off-mass-shell gluon. Another equivalent way is to use the photon wave function formalism [6] which uses the $q\bar{q}$–dipole cross-section where the $q\bar{q}$ configurations are defined by the virtual photon wave function[5]. The target dipole is considered to be small (or massive) in order to justify the (resummed) perturbative QCD calculations.

We shall thus make use of the relation (see Fig. 1 and [9]) between the impact factors and the wave functions [6] of the transverse photon in terms of $q\bar{q}$ configurations for both helicities. The impact factor for $p = 0$, related to the corresponding wave function of the photon reads

$$
\begin{aligned}
V_T^{(p=0)} \frac{C}{\gamma v(1-\gamma)} &= \phi_T^{(p=0)}(\gamma) = \frac{1}{2\pi} \int r \mathrm{d}r \mathrm{d}\varphi \big(r^2 Q^2\big)^{1-\gamma} \\
&\times \int \mathrm{d}z \big(|\Psi_T^+(r,z)|^2 + |\Psi_T^-(r,z)|^2\big), \qquad (18)
\end{aligned}
$$

[4] In the non-perturbative regime, some modifications of the discussion have to be introduced [9] due to the fact that the intermediate gluon g^* may be soft enough to be included in the non-perturbative input. In that case the two pictures lead to two different parametrizations.

[5] The QCD wave function of the photon also has $q\bar{q}g$, $q\bar{q}gg, \ldots,$ components but in the QCD dipole model [16] they are taken into account by the subsequent cascade of dipoles and they contribute to the dipole–dipole cross-section and not to the vertex.

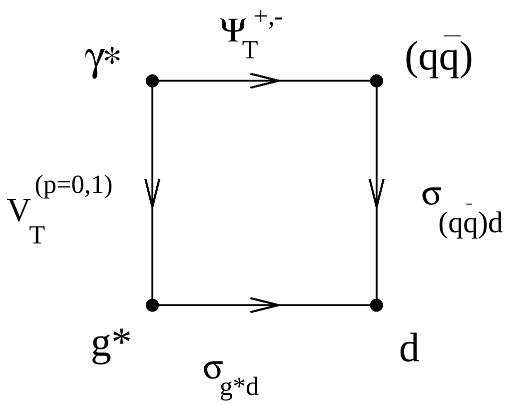

Fig. 1. The two factorization schemes of the γ^*-dipole cross-section. The photon (γ^*)–dipole (d) cross-section, as given by the QCD dipole model corresponding to the perturbative QCD resummation at small x, admits two equivalent factorization schemes (see text). First scheme: $k_{\rm T}$-factorization of the g^*–dipole cross-section with transverse impact factors $V_{\rm T}^{(p=0,1)}$; second scheme: wave function factorization of the $(q\bar{q})$–d cross-section, where the virtual photon transverse wave functions $\Psi_{\rm T}^{(+,-)}$ are described on the basis of its $(q\bar{q})$ configurations. The two conformal spin components ($p = 0, 1$) of the transverse impact factors can be expressed in terms of the wave functions for left (+) and right (-) helicities; see (23), (24)

where

$$v(1-\gamma) \equiv 2^{2\gamma-3} \frac{\Gamma(1+\gamma)}{\gamma(1-\gamma)\Gamma(2-\gamma)}. \tag{19}$$

$v(\gamma)$ is the factorized coupling of the off-mass-shell gluon to a dipole [9]. The light-cone wave functions of the transverse photon $\Psi_{\rm T}^+$ for helicity $^+$ and $\Psi_{\rm T}^-$ for helicity $^-$ are [6]

$$\Psi_{\rm T}^+\left(z, r, Q^2\right) = \sqrt{C}\ z\ {\rm e}^{{\rm i}\varphi} \hat{Q} K_1\left(\hat{Q}r\right), \tag{20}$$

$$\Psi_{\rm T}^-\left(z, r, Q^2\right) = \sqrt{C}\ (1-z)\ {\rm e}^{-{\rm i}\varphi} \hat{Q} K_1\left(\hat{Q}r\right), \tag{21}$$

where K_1 is the Bessel function. By definition $\hat{Q} \equiv Q(z(1-z))^{1/2}$ and the normalization is $C = (\alpha_{\rm em} N_c e^2)/(4\pi\alpha_{\rm s})$.

Now, for an arbitrary combination of both helicities, one finds contributions to two Fourier components in the azimuthal angle (see (14)), namely

$$\begin{aligned} &\left|\eta_+ \Psi_{\rm T}^+ + \eta_- \Psi_{\rm T}^-\right|^2 \\ &\sim \left|\eta_+ z {\rm e}^{{\rm i}\varphi} + \eta_-(1-z){\rm e}^{-{\rm i}\varphi}\right|^2 \hat{Q}^2 K_1^2\left(\hat{Q}r\right) \\ &= \left\{ \underbrace{\eta_+^2 z^2 + \eta_-^2 (1-z)^2}_{(p=0)} \right. \\ &\left. + \underbrace{2\eta_+\eta_- z(1-z)}_{(p=1)} \cos 2\varphi \right\} \hat{Q}^2 K_1^2\left(\hat{Q}r\right). \end{aligned} \tag{22}$$

Normalizing to $\eta_+^2 + \eta_-^2 = 1$, it is easy to realize that the two linearly polarized components $\eta_+ = \pm\eta_- = 1/(2^{1/2})$ give opposite contributions[6] to the component $p = 1$. The coupling to the linearly polarized photon is obtained by inserting the appropriate z-dependent factor in the expression of the wave function contribution to the $p = 1$ component. Projecting on the $p = 1$ azimuthal Fourier component, one writes

$$\begin{aligned} \phi_{\rm T}^{(p=1)}(\gamma) &\equiv \frac{1}{2\pi} \int 2\cos\varphi\ r{\rm d}r{\rm d}\varphi \left(r^2 Q^2\right)^{1-\gamma} \\ &\times \int {\rm d}z 2\Re e\left(\Psi_{\rm T}^+ \Psi_{\rm T}^{-*}\right)(r, z) \\ &= \frac{\alpha_{\rm em} N_c e^2}{4\pi\alpha_{\rm s}} \int r{\rm d}r \left(r^2 Q^2\right)^{1-\gamma} \\ &\times \int {\rm d}z 2z(1-z) \hat{Q}^2 K_1^2(\hat{Q}r) \\ &\sim \int {\rm d}u u^{3-2\gamma} K_1^2(u) \times \int {\rm d}z 2z^\gamma (1-z)^\gamma, \end{aligned}$$

or, noting that the only difference between the two components come from the z-dependent factors,

$$\frac{V_{\rm T}^{(p=1)}}{V_{\rm T}^{(p=0)}} = \frac{\phi_{\rm T}^{(p=1)}}{\phi_{\rm T}^{(p=0)}} = \frac{\gamma}{\gamma+1}. \tag{23}$$

One finally gets

$$\begin{aligned} \begin{pmatrix} V_{\rm T}^{(p=0)} \\ V_{\rm T}^{(p=1)} \end{pmatrix} &\sim \gamma v(1-\gamma) \int \frac{{\rm d}^2 r}{2\pi} \left(r^2 Q^2\right)^{1-\gamma} \\ &\times \int {\rm d}z \begin{pmatrix} z^2 + (1-z)^2 \\ 2z(1-z) \end{pmatrix} \hat{Q}^2 K_1^2(\hat{Q}r), \\ &\sim 2^{-2}(2-\gamma)\ \frac{\Gamma^3(1+\gamma)\Gamma^3(1-\gamma)}{\Gamma(2+2\gamma)\Gamma(4-2\gamma)} \begin{pmatrix} \frac{1+\gamma}{\gamma} \\ 1 \end{pmatrix}. \end{aligned}$$

As we just saw, the perturbative photon couplings to the non-zero conformal spins requires a non-zero linear polarization of the $q\bar{q}$ wave function of the transverse photon to be dynamically active in the reaction. In other terms, the $p = 1$ BFKL component requires a maximal azimuthal correlation while the $p = 0$ one is completely decorrelated azimuthally. Partial azimuthal (de)correlation can be obtained by a mixture of different BFKL components. This will in general depend on the dynamical features of the overall reaction. For instance according to [11], forward jet production in DIS can lead to some azimuthal correlation at small rapidity interval where the higher spin component $p = 1$ still is present. However, the general prediction is a significant azimuthal decorrelation due to the strong dominance of the $p = 0$ component in this case.

In the case of proton structure functions, however, the "sliding mechanism" is able [13] to promote the higher spin components, especially for $p = 1$, to be still important at high energy (and relatively low Q^2) and thus to

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[6] Note an overall sign ambiguity, which has to be fixed by the calculation of both vertices in the process. For instance the overall sign is negative in the forward jet case [11].

keep rather strong azimuthal correlations present in that region. This implies a discussion of the non-perturbative couplings. An important remark is that the "sliding mechanism" is also expected for perturbative couplings when a large ratio exists between the characteristic scales of both vertices. It would thus also be interesting to study such processes where we would predict an increase of the azimuthal correlations accompanying the expected "sliding mechanism".

The non-perturbative couplings, e.g. to the proton, are in general beyond our present theoretical knowledge. This is already true for the leading $p = 0$ conformal components, where there are some ambiguities [9] in the way one is able to factorize the perturbative from the non-perturbative couplings. This is all the more true for the non-leading $p = 1$ component which, to our knowledge, are for the first time studied for proton structure functions in the present paper. For the sake of definiteness, we will follow some reasonable theoretical and phenomenological requirements which we now indicate:
(i) The interaction of the proton are governed (at small x) by its $q\bar{q}$ configurations. This can be interpreted as color dipole configurations [16,9]. Compared with those of the virtual photon, their quantum fluctuations around the proton size Q_0 are expected to be smaller.
(ii) The coupling of the proton will be required to obey the "sliding mechanism", that is to verify the convergence and analyticity properties found in [13]. In particular, no singularity with $\gamma > -1$ should appear in the $p = 1$ coupling.
(iii) Within conditions (i) and (ii), the $p = 0$ (α_2 in (14)) and $p = 1$ (β_2 in (14)) couplings will be assumed to be equal up to a normalization which will be determined phenomenologically and which will represent the necessary degree of *azimuthal correlation* for practical relevance.
(iv) As regards the above-mentioned sign ambiguity of the $p = 1$ vertices: it is removed for the contribution to structure functions which ought to be positive. Thus the product of the photon and proton vertices is considered to be positive.

We shall now propose a convenient class of parametrizations of the proton couplings α_2, β_2 in (13), (14) satisfying the requirements (i)–(iv). Noting [15] the relation

$$\int_0^\infty \mathrm{d}\kappa_0 \frac{\kappa_0^{q-1+2\gamma}}{1+\kappa_0^{2q}} \equiv \frac{1}{2q}\, B\left(\frac{1}{2}+\frac{\gamma}{q}, \frac{1}{2}-\frac{\gamma}{q}\right) = \frac{\pi}{2q\cos\left(\frac{\gamma}{q}\right)}, \tag{24}$$

we are led to choose

$$\alpha_2, \beta_2 \propto \frac{\kappa_0^{(q+2)}}{1+\kappa_0^{2q}}; \quad f_0(\gamma), f_1(\gamma) \propto \frac{1}{\cos\left(\frac{\gamma}{q}\right)}\ . \tag{25}$$

Eventually, one may vary the peaking of the distribution around $\kappa_0 = 1$ by changing the values of q. It is interesting to note that for $q \geq 2$ the gauge invariance constraint [2] $\alpha_2(0) = \beta_2(0) = 0$ is automatically verified. One can also multiply by a polynomial expression in γ. This can be used to satisfy the constraints, in particular the analyticity ones by cancelling the poles at $\gamma > -1$.

4 The two-pomeron conjecture and azimuthal matching

As already mentionned, the non-zero conformal spin components are generally neglected in the phenomenology related to the BFKL equation. Indeed, at ultra-high energy $Y \to \infty$, the structure function components in (3) are driven by the saddle-points at $\gamma = 1/2$. It is easy to realize that the corresponding intercepts $\chi_p\,(1/2)$ are all negative for $p \neq 0$. At the same time their effective anomalous dimension $1/2$ means that they all contribute to a leading-twist behavior. However, it has been remarked in [13] that at large but finite values of Y or Q^2 the corresponding saddle-points *slide* away from $\gamma = 1/2$ and generate contributions with very different Y and Q^2 behavior from the ultra-asymptotic ones. In particular the $p = 1$ component is still increasing with energy (positive intercept) and their Q^2 behavior mimicks a higher-twist behavior, i.e. they decrease with a negative power of Q^2. Both features allowed the authors of [13] to look for the possibility that the $p = 1$ component could be interpreted as the high Q^2 remnant of the "soft" pomeron considered as an higher-twist contribution from the point of view of the operator product expansion of QCD. This would provide a QCD framework for the two-pomeron hypothesis proposed in [14] to describe the phenomenological features of structure functions in a different, Regge approach.

Let us now investigate how the phenomenological discussion can be influenced by the determination of the conformal couplings derived in the previous sections. In order to analyze the phenomenology of structure functions in a manner similar to [13,14], we have to introduce our determination (24) of the perturbative coupling to the photon and discuss the proton coupling, using, for instance, the family of parametrizations (25). In the discussion, however, it is important to take into account the ambiguity of the separation between perturbative and non-perturbative couplings discussed in [9] for the $p = 0$ component. Let us recall the problem and extend its lessons to the $p = 1$ component.

We will consider the following parametrization[7] of the functions f_p to be inserted in (3):

$$f_0(\gamma) = \phi_{\mathrm{T}}^{(p=0)}(\gamma) \times \frac{\gamma(\gamma+1)}{\cos\frac{\pi\gamma}{q}}. \tag{26}$$

$$f_1(\gamma) = \phi_{\mathrm{T}}^{(p=1)}(\gamma) \times \mathcal{N}_I \frac{\gamma(\gamma+1)}{\cos\frac{\pi\gamma}{q}}, \tag{27}$$

[7] In [9], two different models were introduced, depending on whether the factorization between perturbative and non-perturbative couplings is assumed at the intermediate gluon level (model I in [9]) or at the quark level (model II in [9]). This ambiguity relies on the possibility of the gluon coupling to the $q\bar{q}$ configurations of the photon (with its typical singularity in $1/\gamma$) present (model I) or absorbed (model II) in the non-perturbative coupling to the proton. We checked that the results we obtained in the framework of model I are very similar for model II up to a renormalization of the $p = 1$ component.

where one considers the product of a perturbatively determined factor ϕ_T and a non-perturbative factor which has to be modeled. The non-perturbative coupling has been chosen in order to satisfy the analyticity and convergence constraints in a minimal way. Assuming the same analytic form for the non-perturbative $p = 1$ proton coupling for $p = 0$, the arbitrary normalization $\mathcal{N}_I$ quantifies the relative weight which we want to evaluate. The value $q = 4$ has been choosen for convenience. $q > 2$ at least is needed to verify the constraint (ii). We checked that the results are rather independent of these choices, provided the constraints are satisfied. Note that f_1 is "softer" at $\gamma = 0$ than f_0 due to the relative factor $\gamma/(\gamma + 1)$.

From a physical point of view, the non-perturbative factors in formulae (26), (27) can be interpreted [17,9] as related to the wave functions of the *primordial dipole* configurations in the proton. In the QCD dipole model [16] the BFKL dynamics can be expressed in terms of the dipole–dipole cross-section. Translating this model in the case of γ^*–proton scattering, it amounts to considering this cross-section averaged both over the $q\bar{q}$ configurations of the photon and the *primordial dipole* configurations of the proton.

In order to check[8] the sliding mechanism advocated in [13], we display in Fig. 2 the normalization independent plot $(\partial \ln F_p)/(\overline{\alpha}\partial Y)$ as a function of $(\partial \ln F_p)/(\partial \ln Q^2)$ for large Y and different values of $\ln Q^2/Q_0^2$. On the same plot and for the same values is also shown the corresponding results for the Regge parametrization of [14]. As discussed in [13], the results (the black circles in Fig. 1) gives the location in a two-dimensional representation where the effective intercept is plotted as a function of the effective saddle-point γ_c. They can be shown [13] to be situated near the curves defined by the functions $\epsilon_p(\gamma)$, independently of the peculiar form of the factors $f_{0,1}$. The sizable sliding of the $p = 1$ component is proven by the shift of the corresponding points with respect to the ultra-asymptotic value at $\gamma_c = 1/2$. Moreover, the evolution at large Q^2 meets the phenomenological determination of the two pomeron components of [14] for $\log\left(Q^2/Q_0^2\right) \sim 8, 10$ and reasonable values of the parameter $\bar{\alpha} \sim 0.4$.

In order to determine the relative strength of the $p = 1$ and $p = 0$ components, and thus the rôle of the conformal prefactors $f_{0,1}$ (see formulae (26), (27)), we have considered the two-pomeron fit ("hard" and "soft") of [14] in the large Q^2 region where it meets[9] the behavior of the two ($p = 0$ and $p = 1$) conformal spin components. For instance we show in Fig. 3 the Y dependence at fixed large Q^2.

The results indicate large normalizations, namely $\mathcal{N}_I \sim 50$. We have checked that the normalization remains of the same order with other convenient non-perturbative ansätze. Thus, the two-pomeron conjecture (as seen from a QCD point of view) is obtained only if a strong dynamical enhancement favors the non-perturbative coupling of the $p = 1$ component. Since the perturbative coupling is maximal for a linearly polarized transverse photon and limited in size because of positivity constraints (the photon has only $q\bar{q}$ configurations; see footnote 5) the relevance of the $p = 1$ coupling relies on the existence of a non-perturbative mechanism enhancing considerably the

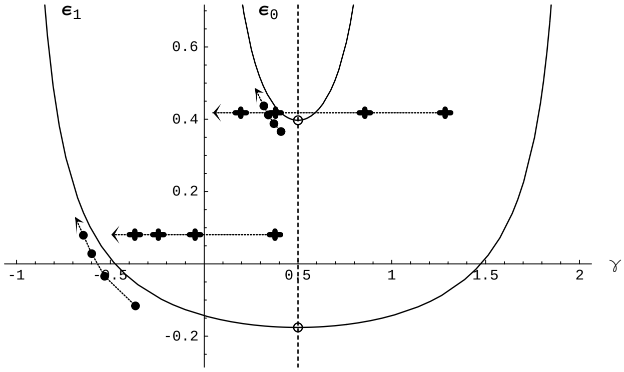

Fig. 2. Plot of effective intercept vs. effective dimension at fixed large Y. The effective intercept $\partial \ln F_{0,1}/\bar{\alpha}\partial Y$ plotted vs. the effective anomalous dimension $\partial \ln F_{0,1}/\partial \ln Q^2$ is compared to the functions $\epsilon_{0,1} \equiv \bar{\alpha}\chi_{0,1}(\gamma)$ (see (1), (3)). They are computed at $\bar{\alpha} = 0.15$ for fixed $Y = 10$ and 4 values of $\ln Q^2/Q_0^2 = \{4, 6, 8, 10\}$. The weight in the integrals (3) corresponds to (26), (27). Black circles: numerical results; white circles: ultra-asymptotic saddle points at $\gamma = 1/2$; full lines: the functions $\epsilon_p(\gamma)$ for $(p = 0, 1)$; crosses; results from the Regge fit of [14] corresponding to the same value of Y and $\ln Q^2/Q_0^2$, with $Q_0 \sim 200\,\mathrm{MeV}$. Arrows indicate the direction of increasing Q

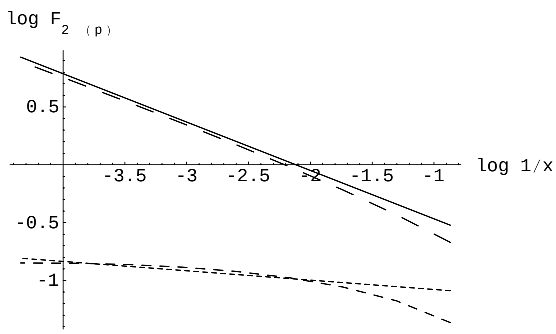

Fig. 3. The structure function spin components $F_{2(p=0,1)}$ at fixed large Q^2. The structure function components $F_{2(p=0,1)}$ are displayed as a function of $Y = \log 1/x$ and compared with the parametrization of the two-pomeron model of [14] at $Q^2 = 1000\,\mathrm{GeV}^2$. This value is choosen to correspond to $\ln Q^2/Q_0^2 \sim 10$. Continuous line: "hard pomeron" component of [14]; long-dashed line: spin 0 component. Short-dashed line: "soft pomeron" component of [14]; dashed line: spin-1 component

[8] We used the model I parametrization, but the results are the same for model II or by changing $q > 2$.

[9] This comparison is to be taken only with a grain of salt since it is made in a region where the "soft" component is weak and thus not directly determined by data. A determination at small Q^2 would be more precise, but then the non-perturbative corrections are expected to be important and may invalidate a correct evaluation of the normalization in a BFKL framework.

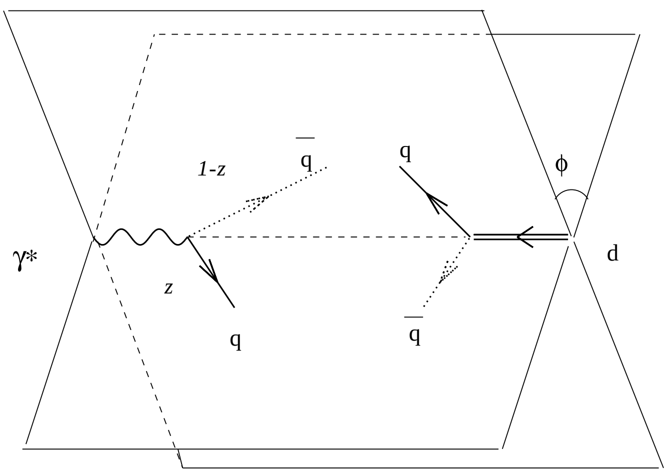

Fig. 4. Azimuthal matching of photon and dipole $q\bar{q}$ configurations. The photon (γ^*)–dipole (d) reaction is represented in the center-of-mass frame. The azimuthal angle between $q\bar{q}$ configurations of both colliding systems is the angle ϕ between the two planes. The quark (respectively antiquark) momentum fraction in the virtual photon is z (respectively $1-z$) (the similar variable for the dipole configurations has been averaged already)

matching between the proton primordial dipole configurations and the azimuthal polarization of the virtual photon.

We shall now speculate on such a non-perturbative mechanism based on azimuthal matching in γ^*–proton scattering.

The mechanism is the following (see Fig. 4 for a schematic representation). It has been known since a long time [6] that deep-inelastic lepton proton scattering is not necessarily dominated by a "hard" process if the energy is large with respect to the photon virtuality, e.g. if $x \sim Q/W$ is small. Indeed the *effective* virtuality is $\hat{Q} = Q(z(1-z))^{1/2}$, where z (respectively $1-z$) is the momentum fraction of the quark (resp. antiquark) in the virtual $q\bar{q}$ state configurations of the virtual photon. This is explicit, for instance, in the wave functions (21). Thus, if the favored $q\bar{q}$ configurations are particularly assymetric (aligned jet [6] configurations) one even may reach the situation where the quark or the antiquark in the pair has such a small momentum that $\hat{Q}$ is of order unity and the reaction is dominated by a "soft" process.

However, the experimental results de not seem to favor the aligned jet mechanism since a "hard" component shows up which is precisely the one which could be described by the $p = 0$ component. Yet, for the $p = 1$ component, it is not excluded that a partial jet alignment can take place, at least at moderate Q^2. At high Q^2 one could then expect that asymmetric configurations are substantially favored in a kind of "hard/soft" compromise: the effective virtualities $\hat{Q}$ are smaller than Q while remaining in the "semi-hard" regime. As a consequence, one expects a substantial *azimuthal matching* between the $q\bar{q}$ configurations of the virtual photon and the $q\bar{q}$ (or primary dipole [17]) configurations in the proton; see Fig. 3. This *azimuthal matching* may give a strong dynamical enhancement for the coupling of the linearly polarized components of the photon to the proton. By this azimuthal enhancement one could find the qualitative justification for the two-pomeron description to be based on the two conformal spin components. On the other hand, in the absence of such a mechanism the $p = 1$ components, even if increasing with energy due to the sliding phenomenon, would not be coupled enough to the proton to give rise to a sizable component. We shall in conclusion discuss possible tests of azimuthal alignment which is certainly deserving further study.

5 Summary and conclusions

Let us briefly summarize our results:
(i) Taking into account that the non-zero (indeed the $p = 1$) conformal spin components of the BFKL QCD pomeron could be phenomenologically relevant in deep-inelastic lepton proton scattering, we have given the formal expression of the proton structure functions' conformal spin components in terms of the appropriate *impact factors*.
(ii) We have computed the perturbative *impact factor* for the $p = 1$ component at the virtual photon vertex using a general relation between impact factors and $q\bar{q}$ wave functions. The key result is that the coupling is maximal for linear azimuthal polarization and zero for circular (or no) polarization.
(iii) In order to be phenomenologically relevant as a "second" pomeron contribution in γ^*–proton scattering, a strong *azimuthal matching* with the primordial dipole $q\bar{q}$ configurations of the proton is required. This non-perturbative mechanism could be associated in part with jet alignment *à la Bjorken* for the $p = 1$ component, while it is expected to be weak or absent for the $p = 0$ one.

Some comments are in order. The large enhancement (a factor ~ 50) of the non-perturbative coupling to the $p = 1$ component that we found necessary to match with the two-pomeron parametrization of [14] is consistent with the key point of this kind of phenomenological analysis: the mismatch between the "hard" and "soft" pomerons is more important than in models with only one effective pomeron singularity. In particular, at intermediate values of the virtuality Q^2, both contributions are important. This is the reason why the "hard" component has a large intercept $\epsilon\,(p = 0, 1/2) \simeq 0.4$ in agreement with the theoretical range of values, and larger than the effective intercept $\epsilon\,(p = 0, 1/2) \leq 0.3$; see for instance [17,9]. The validity of a non-negligible "soft" pomeron coupling at high Q^2 is thus to be checked in further study.

A specific feature of the $p = 1$ component is its special azimuthal properties. The question arises whether it is possible to isolate it using azimuthal correlation properties. For the total inclusive process, leading to the determination of the structure function itself, this does not seem easy. The relevant azimuthal axis in the photon–dipole center-of-mass frame, see Fig. 4, can be very different from the photon–proton one, and thus, in particular, the s-channel helicity conservation which seems to be an approximate property of the "soft" pomeron coupling is not in contradiction with the conformal spin properties of the $p = 1$ component.

A possible test of the *azimuthal matching* could be performed in forward jet production in deep inelastic scattering. Indeed, while the commonly considered configuration with similar scales for the photon probe and the jet is expected to lead to a small azimuthal correlation in a high rapidity interval [11], the case with a larger scale ratio[10] is expected to lead to stronger azimuthal correlation due to the enhancement with energy of the higher conformal spin component responsible for the azimuthal matching in the considered formalism.

Indeed, a practical way of checking the azimuthal correlations could be to fix a certain range of high[11] Q^2 for the photon virtuality and vary the tranverse momentum of the forward jet down to the lower admissible value to select a jet. In this way, one increases the ratio Q^2/k_{T}^2 of scales and thus enhances the energy behavior of the $p = 1$ component. Moreover, since the model implies a strong mismatch between the "soft" and "hard" pomerons at intermediate scale, one expects the development of a stronger (and perhaps different in sign!) azimuthal correlation than the perturbatively predicted azimuthal correlations (with negative sign) studied in [11]. A similar method can be proposed at Tevatron analyzing azimuthal correlations between two jets (1) and (2) in different hemispheres, as analyzed in [10], with the prediction that it will increase together with the ratio $k_{\mathrm{T}}^{(1)}/k_{\mathrm{T}}^{(2)}$.

On theoretical grounds, it would be useful to use the eigenvectors and eigenvalues of the conformal invariant part (or of the effective conformal invariant form of) the evolution kernels at next-leading levels to analyze the sliding mechanism in this context. However, the known complications (due to the mismatch at all orders of the perturbation expansion with the renormalization group evolution) are to be clarified before entering into more specific calculations. This deserves a new study in the future.

[10] It is useful to note that, in this case of rather large scale ratio, the generalized BFKL formula (3) leads to an interesting connection with higher-twist terms in the operator product expansion of the structure function. Indeed, while the leading conformal spin component becomes identical to the double-log approximation of the DGLAP equation, it is not difficult to realize that higher conformal spin components correspond to higher-twist contributions. The *sliding mechanism*, if confirmed, could allow one in this case to investigate the evolution of higher-twist contributions with the scale ratio generalizing the double-log approximation of the DGLAP equation. This connection certainly deserves a specific study which is beyond the scope of our paper, since it is a non-trivial extension of the BFKL formalism.

[11] However, in practice Q^2 is limited by the necessity of a large rapidity interval with the forward jet.

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